Discrete Mathematics

CMP-200

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Lecture 2

Text Book

Text Book(s)	Kenneth H. Rosen "Discrete Mathematics and Its Applications", 6^{th} Ed., Mc. Graw Hilll, 2007.

Assessment Criteria

Sessional 25%		Mid 35%	Final 40%
Quizzes	25	Written Exam 35	Written Exam 40
Total	25	35	40
	100		

Converse, Inverse and Contrapositive

- Converse
  - The converse of a conditional is formed by switching the hypothesis and the conclusion.
  - The converse of $p \rightarrow q$ is $q \rightarrow p$
- Inverse
  - Negate the hypothesis and the conclusion
  - The inverse of $p \rightarrow q$, is $\neg p \rightarrow \neg q$
- Contrapositive
  - Negate the hypothesis and the conclusion of the converse
  - The contrapositive of $p \rightarrow q$, is $\neg q \rightarrow \neg p$.

Example

- Write the (a) inverse, (b) converse, and (c) contrapositive of the statement.
  - If two angles are vertical, then the angles are congruent.
- (a) Inverse: If 2 angles are not vertical, then they are not congruent.
- (b) Converse: If 2 angles are congruent, then they are vertical.
- (c) Contrapositive: If 2 angles are not congruent, then they are not vertical.

Converse, Inverse, Contrapositive

Consider the proposition $p \rightarrow q$

- Its converse is the proposition $q \rightarrow p$
- Its inverse is the proposition $\neg p \rightarrow \neg q$
- Its contrapositive is the proposition $\neg q \rightarrow \neg p$

Constructing Truth Tables

- Construct the truth table for the following compound proposition

$$((p \wedge q) \vee \neg q)$$

p	q	$p \wedge q$	$\neg q$	$((p \wedge q) \vee \neg q)$
0	0	0	1	1
0	1	0	0	0
1	0	0	1	1
1	1	1	0	1

Exercise on Logical Connectives

Negation

What is the negation of each of these propositions?

- Today is Wednesday.
- There is no pollution in Lahore.
- 2 + 1 = 3
- The summer in Karachi is hot and sunny.

Exercise on Logical Connectives

Multiple Connectives.

Let p and q be the propositions

p : I bought a lottery ticket this week.

q : I won the million dollar jackpot on Friday.

Express each of the next propositions as an English sentence.

Exercise on Logical Connectives

p : I bought a lottery ticket this week.

q : I won the million dollar jackpot on Friday.

$\neg p$

I did not buy a lottery ticket this week.

Exercise on Logical Connectives

p : I bought a lottery ticket this week.

q : I won the million dollar jackpot on Friday.

$$p \vee q$$

Either I bought a lottery ticket this week or I won the million dollar jackpot on Friday.

Exercise on Logical Connectives

p : I bought a lottery ticket this week.

q : I won the million dollar jackpot on Friday.

$$p \rightarrow q$$

If I bought a lottery ticket this week, then I won the million dollar jackpot on Friday.

Exercise on Logical Connectives

p : I bought a lottery ticket this week.

q : I won the million dollar jackpot on Friday.

$$p \wedge q$$

I bought a lottery ticket this week and I won the million dollar jackpot on Friday.

Exercise on Logical Connectives

p : I bought a lottery ticket this week.

q : I won the million dollar jackpot on Friday.

$$p \leftrightarrow q$$

I bought a lottery ticket this week if and only if I won the million dollar jackpot on Friday

Exercise on Logical Connectives

p : I bought a lottery ticket this week.

q : I won the million dollar jackpot on Friday.

$$\neg p \rightarrow \neg q$$

If I did not buy a lottery ticket this week, then I did not win the million dollar jackpot on Friday.

Exercise on Logical Connectives

p : I bought a lottery ticket this week.

q : I won the million dollar jackpot on Friday.

$$\neg p \wedge \neg q$$

I did not buy a lottery ticket this week, and I did not win the million dollar jackpot on Friday.

Exercise on Logical Connectives

p : I bought a lottery ticket this week.

q : I won the million dollar jackpot on Friday.

$$\neg p \lor (p \land q)$$

Either I did not buy a lottery ticket this week, or else I did buy one and won the million dollar jackpot on Friday.

Translating English Sentences

Why?

English (and every other human language) is often ambiguous. Translating sentences into compound statements (and other types of logical expressions, removes the ambiguity.

Translating English Sentences

This may involve making a set of reasonable assumptions based on the intended meaning of the sentence.

once we have translated sentences from English into logical expressions we can analyze these logical expressions to determine their truth values, we can manipulate them.

Example 1

How can this English sentence be translated into a logical expression?

"You can access the Internet from campus only if you are a computer science major or you are not a freshman."

Example 1

It is possible to represent the sentence by a single propositional variable, such as p

It will not be beneficial when we are to analyze the algorithm

What may be the solution?

Example 1

Let a="You can access the Internet from campus"

c="You are a computer science major“

f="You are a freshman" respectively. Noting that "only if" is one way a conditional statement can be expressed, this sentence can be represented as

$$a \rightarrow (c \vee \neg f).$$

Example 2

Let p and q be the propositions

p : It is below freezing.

q : It is snowing.

Write the propositions using p and q and logical connectives.

Example 2

p : It is below freezing.

q : It is snowing.

It is below freezing and snowing.

$$p \wedge q$$

Example 2

p : It is below freezing.

q : It is snowing.

It is below freezing but not snowing.

$$p \land \neg q$$

Example 2

p : It is below freezing.

q : It is snowing.

It is not below freezing and it is not snowing

$$\neg p \wedge \neg q$$

Example 2

p : It is below freezing.

q : It is snowing.

It is either snowing or below freezing (or both)

$$p \vee q$$

Example 2

p : It is below freezing.

q : It is snowing.

If it is below freezing, it is also snowing.

$$p \rightarrow q$$

Example 2

p : It is below freezing.

q : It is snowing.

- It is either below freezing or it is snowing, but it is not snowing if it is below freezing.

$$(p \vee q) \wedge (p \rightarrow \neg q)$$

Example 2

p : It is below freezing.

q : It is snowing.

That it is below freezing is necessary and sufficient for it to be snowing

$$p \leftrightarrow q$$

Example 2

p : It is below freezing.

q : It is snowing.

That it is below freezing is necessary and sufficient for it to be snowing

$$p \leftrightarrow q$$

Example 3

Let p, q, and r be the propositions

p : You have the flu.

q : You miss the final examination.

r : You pass the course.

Express each of these propositions as an English sentence

Example 3

p : You have the flu.

q : You miss the final examination.

r : You pass the course.

$$p \rightarrow q$$

If you have the flu, then you miss the final exam.

Example 3

p : You have the flu.

q : You miss the final examination.

r : You pass the course.

$$\neg q \leftrightarrow r$$

You do not miss the final exam if and only if you pass the course

Example 3

p : You have the flu.

q : You miss the final examination.

r : You pass the course.

$$q \rightarrow \neg r$$

If you miss the final exam, then you do not pass the course

Example 3

p : You have the flu.

q : You miss the final examination.

r : You pass the course.

$$p \vee q \vee r$$

You have the flu, or miss the final exam, or pass the course

Example 3

p : You have the flu.

q : You miss the final examination.

r : You pass the course.

$$(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$$

It is either the case that if you have the flu then you do not pass the course or the case that if you miss the final exam then you do not pass the course (or both, it is understood)

Example 3

p : You have the flu.

q : You miss the final examination.

r : You pass the course.

$$(p \wedge q) \vee (\neg q \wedge r)$$

Either you have the flu and miss the final exam, or you do not miss the final exam and do pass the course.

Logical Puzzle

Puzzles that can be solved using logical reasoning are known as logic puzzles.

Solving logic puzzles is an excellent way to practice working with the rules of logic.

Also, computer programsdesigned to carry out logical reasoning often use well-known logic puzzles to illustrate their capabilities.

Raymond Smullyan, a master of logic puzzles

- Knights always tell the truth, and knaves always lie
- A says “B is a knight”
- B says “The two of us are opposite types”

What are A and B?

Muddy Children Puzzle

- A boy and girl are sent to play by their father
- Both are ordered not to get dirty.
- Both get mud on their forehead, however.
- Father says "At least one of you has a muddy forehead,"

"Do you know whether you have a muddy forehead?" Asks twice