PUNJAB UNIVERSITY CENTER FOR INFORMATION TECHNOLOGY

UNIVERSITY OF THE PUNJAB, LAHORE

Q:1: Let p, q , and r be the propositions

p : Grizzly bears have been seen in the area.

q : Hiking is safe on the trail.

r : Berries are ripe along the trail.

Write these propositions using p, q, and r and logical connectives.

a) Berries are ripe along the trail, but grizzly bears have not been seen in the area.

$$r \wedge \neg p$$

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b) Grizzly bears have not been seen in the area and hiking on the trail is safe, but berries are ripe along the trail.

$$\neg p \wedge q \wedge r$$

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c) If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area.

$$r \rightarrow (q \leftrightarrow \neg p)$$

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d) It is not safe to hike on the trail, but grizzly bears have not been seen in the area and the berries along the trail are ripe.

$$\neg q \wedge \neg p \wedge r$$

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e) For hiking on the trail to be safe, it is necessary but not sufficient that berries not be ripe along the trail and for grizzly bears not to have been seen in the area.

$$(q \rightarrow (\neg r \wedge \neg p)) \wedge \neg((\neg r \wedge \neg p) \rightarrow q)$$

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f) Hiking is not safe on the trail whenever grizzly bears have been seen in the area and berries are ripe along the trail.

$$(p \wedge r) \rightarrow \neg q$$

Q:2: Show that the $(p \rightarrow q) \wedge (p \rightarrow r)$ *and* $p \rightarrow (q \wedge r)$ are logically equivalent.

p	q	r	$p \rightarrow q$	$p \rightarrow r$	$(p \rightarrow q) \wedge (p \rightarrow r)$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	F
T	F	F	F	F	F
F	T	T	T	T	T
F	T	F	T	T	T
F	F	T	T	T	T
F	F	F	T	T	T

p	q	r	$q \wedge r$	$p \rightarrow (q \wedge r)$
T	T	T	T	T
T	T	F	F	F
T	F	T	F	F
T	F	F	F	F
F	T	T	T	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

Q:3: Let *C (x)* be the statement "x has a cat," let *D (x)* be the statement "x has a dog," and let *F(x)* be the statement "x has a ferret." Express each of these statements in terms of *C(x), D(x), F(x),* quantifiers, and logical connectives. Let the domain consist of all students in your class.

a) A student in your class has a cat, a dog, and a ferret.

$$\exists x(C(x) \wedge D(x) \wedge F(x))$$

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b) All students in your class have a cat, a dog, or a ferret.

$$\forall x(C(x) \vee D(x) \vee F(x))$$

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c) Some student in your class has a cat and a ferret, but not a dog.

$$\exists x(C(x) \wedge F(x) \wedge \neg D(x))$$

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d) No student in your class has a cat, a dog, and a ferret.

$$\neg\exists x(C(x) \wedge D(x) \wedge F(x))$$

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e) For each of the three animals, cats, dogs, and ferrets, there is a student in your class who has one of these animals as a pet.

$$(\exists x\, C(x)) \wedge (\exists x\, D(x)) \wedge (\exists x\, F(x))$$

Q:4: P (x , y) be the statement "student x has taken class y," where the domain for x consists of all students in your class and for y consists of all computer science courses at your school. Express each of these quantifications in English.

a) $\exists x\, \exists y\, P(x, y)$

Some student in your class has taken some computer science course.

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b) $\exists x\, \forall y\, P(x, y)$

There is a student in your class who has taken every computer science course.

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e) $\forall x\, \exists y\, P(x, y)$

Every student in your class has taken at least one computer science course.

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d) $\exists y\ \forall x\, P(x, y)$

There is a computer science course that every student in your class has taken.

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e) $\forall y\, \exists x\, P(x, y)$

Every computer science course has been taken by at least one student in your class.
